MTH 304: Metric Spaces and Topology

Practice Assignment I : Quotient topology

1. Establish the assertions in 1.10 (vi) of the lesson plan.

2. Establish in the assertions in examples 1.10 (viii) (b), (c), and (xii) (b).

3. Show that the following maps are quotient maps.

   (a) A continuous map
   $$f : X \to Y$$
   which has a right inverse (i.e $\exists$ a $g : Y \to X$ such that $f \circ g = i_Y$).

   (b) For a subspace $A \subset X$, a continuous map
   $$r : X \to A$$
   such that
   $$r|_A = i_A.$$
   Such a map is called a *retraction*.

   (c) The projection map
   $$\pi_j : \prod_{i=1}^{n} X_i \to X_j$$
   onto the jth coordinate.

4. Describe the following quotient spaces on $X = \mathbb{R}^2$.

   (a) $X/\sim$, where $(x_1, y_1) \sim (x_2, y_2) \iff x_1 + y_1^2 = x_2 + y_2^2$.
   (b) $X/\sim$, where $(x_1, y_1) \sim (x_2, y_2) \iff x_1^2 + y_1^2 = x_2^2 + y_2^2$.